1.

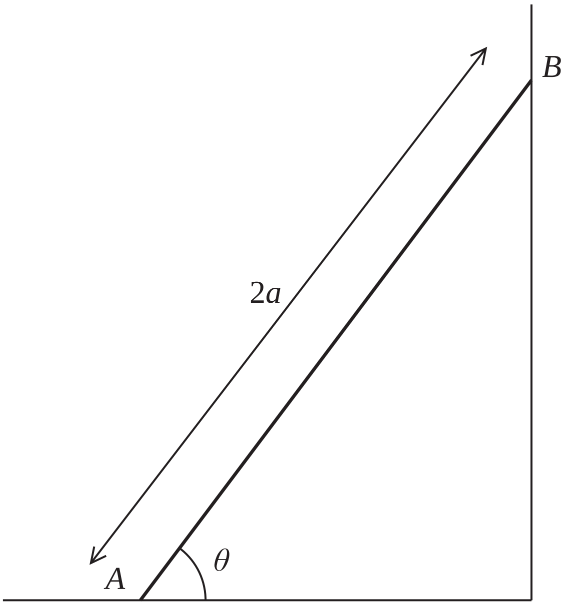


Figure 2

A beam AB has mass m and length $2a$.

The beam rests in equilibrium with A on rough horizontal ground and with B against a smooth vertical wall.

The beam is inclined to the horizontal at an angle θ, as shown in Figure 2.

The coefficient of friction between the beam and the ground is μ

The beam is modelled as a uniform rod resting in a vertical plane that is perpendicular to the wall.

Using the model,

(a) show that $\mu \geqslant \frac{1}{2}\cot\theta$

(5)

A horizontal force of magnitude kmg, where k is a constant, is now applied to the beam at A.

This force acts in a direction that is perpendicular to the wall and towards the wall.

Given that $\tan\theta = \frac{5}{4}$, $\mu = \frac{1}{2}$ and the beam is now in limiting equilibrium,

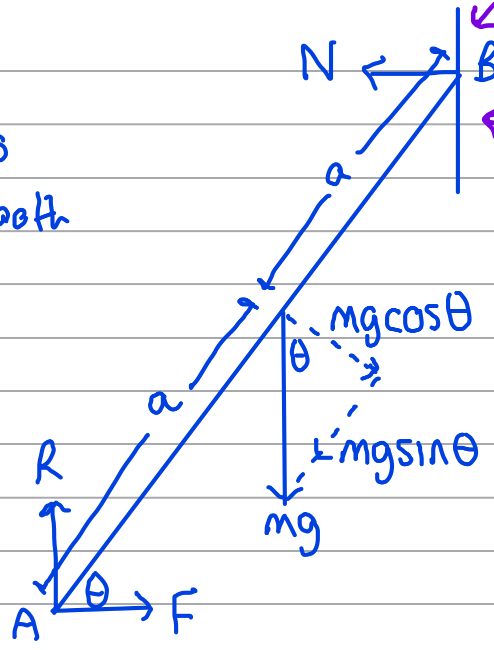
(b) use the model to find the value of k.

(5)

Reaction force perpendicular to wall B.

reaction force perp to wall B
no friction, smooth

smooth wall, no friction



a) Beam does not slip, so $F \leq \mu R$

Method: find F and R

$R(\uparrow): R = mg$ 1 $\quad R(\rightarrow): F = N$

$M(A):$ 1 $amg\cos\theta = 2aN\sin\theta$ 1

"Moments about A"

$\Rightarrow N = \dfrac{amg\cos\theta}{2a\sin\theta}$

force × perpendicular distance

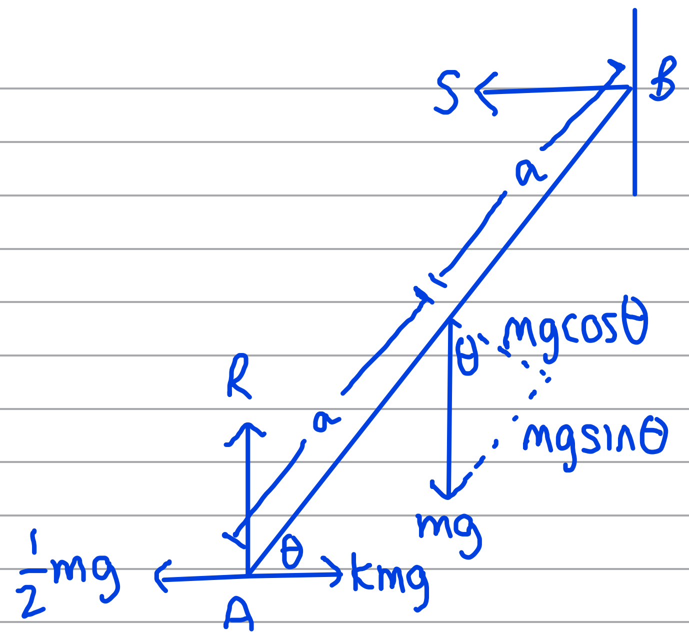
$N = \frac{1}{2}mg\cot\theta$

$F = N \quad \therefore F = \frac{1}{2}mg\cot\theta$

$F \leq \mu R \therefore \mu \geq \dfrac{F}{R} \Rightarrow \mu \geq \dfrac{\frac{1}{2}mg\cot\theta}{mg}$ 1

$\mu \geq \frac{1}{2}\cot\theta$ 1

b)



$\tan\theta = \frac{5}{4}$

$\therefore \cot\theta = \frac{4}{5}$

Limiting equilibrium $\therefore F = \mu R$

$\mu = \frac{1}{2} \therefore F = \frac{1}{2}mg$

Friction acts to the left as the beam is on the verge of slipping to the right

$R(\uparrow): R = mg$ $\quad R(\rightarrow): S + \frac{1}{2}mg = kmg$ (1)

(1) $\qquad S = (k - \frac{1}{2})mg$

$M(A): amg\cos\theta = 2Sa\sin\theta$ (1)

$\Rightarrow S = \frac{1}{2}mg\cot\theta = \frac{1}{2}mg\left(\frac{4}{5}\right) = \frac{2}{5}mg$

$\therefore k - \frac{1}{2} = \frac{2}{5}$ (1)

$k = 0.9$ (1)

2.

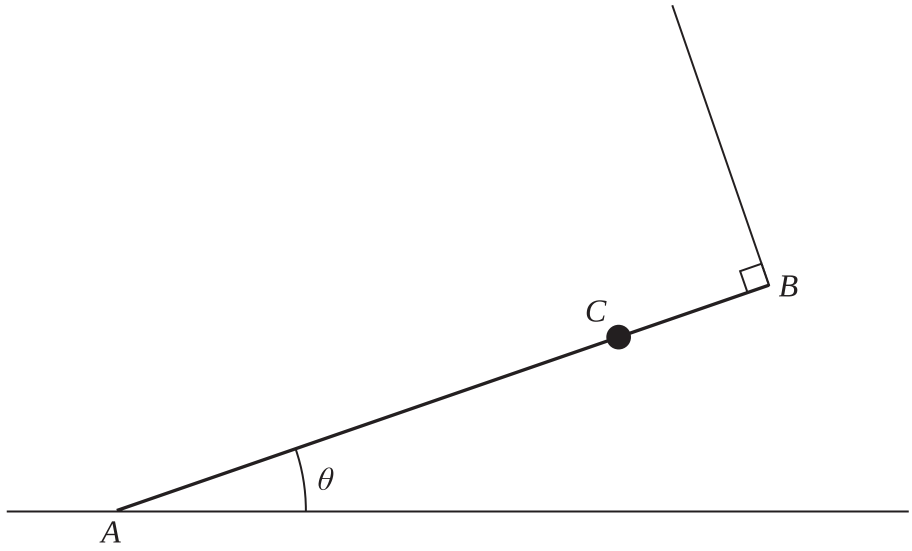


Figure 2

A uniform rod AB has mass M and length $2a$

A particle of mass $2M$ is attached to the rod at the point C, where $AC = 1.5a$

The rod rests with its end A on rough horizontal ground.

The rod is held in equilibrium at an angle θ to the ground by a light string that is attached to the end B of the rod.

The string is perpendicular to the rod, as shown in Figure 2.

(a) Explain why the frictional force acting on the rod at A acts horizontally to the right on the diagram.

(1)

The tension in the string is T

(b) Show that $T = 2Mg\cos\theta$

(3)

Given that $\cos\theta = \frac{3}{5}$

(c) show that the magnitude of the vertical force exerted by the ground on the rod at A is $\frac{57Mg}{25}$

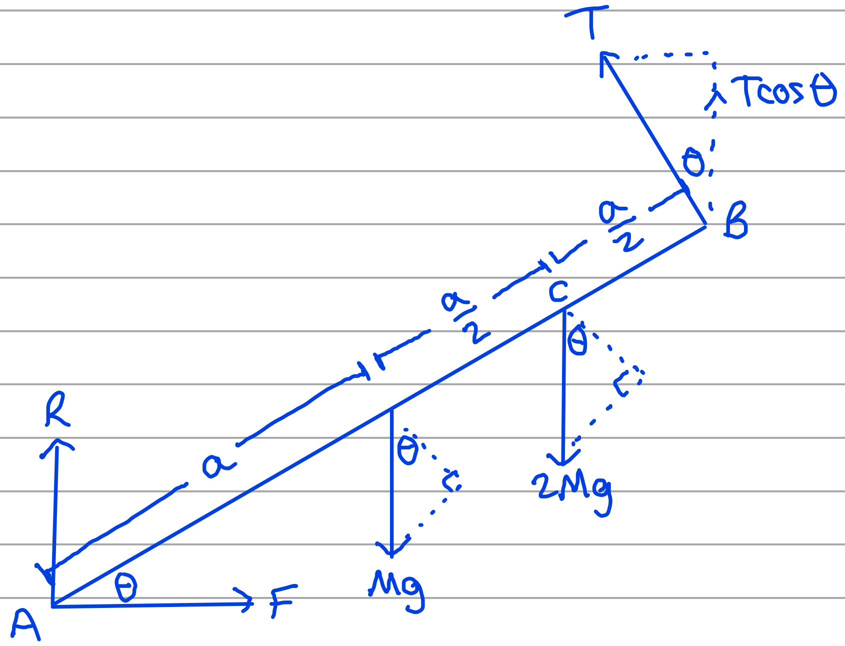
(3)

The coefficient of friction between the rod and the ground is μ

Given that the rod is in limiting equilibrium,

(d) show that $\mu = \frac{8}{19}$

(4)



a) The only other force that has a horizontal component is T, which acts to the left. For the rod to be in equilibrium, friction must act to the right. (1)

b) $M(A): Mg \times a\cos\theta + 2Mg \times \frac{3a}{2}\cos\theta = T \times 2a$ (1) (1)

$$4aMg\cos\theta = 2aT$$

$$T = \frac{4aMg\cos\theta}{2a} = 2Mg\cos\theta \quad (1)$$

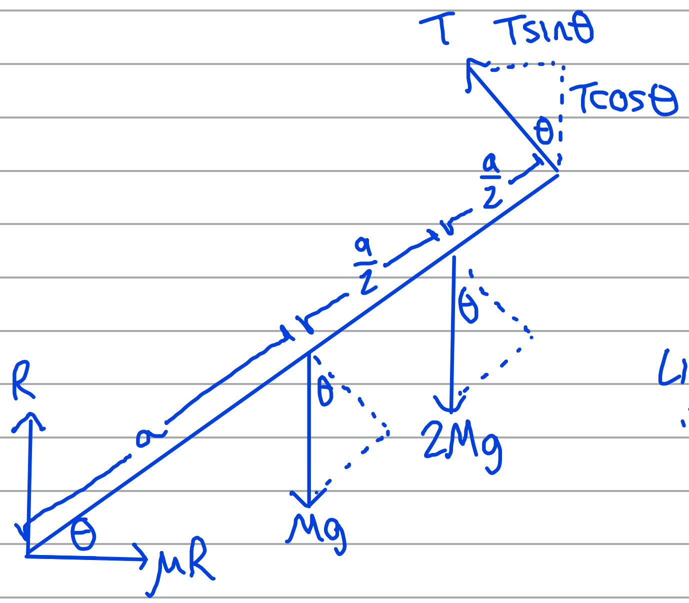
c) $\cos\theta = \frac{3}{5}$ $\quad \therefore T = 2Mg\left(\frac{3}{5}\right) = \frac{6Mg}{5}$ (1)

$R(\uparrow): R + T\cos\theta = Mg + 2Mg$ (1)

$$R = 3Mg - \frac{6}{5}Mg\left(\frac{3}{5}\right)$$

$$R = \frac{57Mg}{25} \quad (1)$$

d)



Limiting equilibrium
$\therefore F = \mu R$ (1)

(1)
$R(\rightarrow)$: $\mu R = T\sin\theta$ (1)

sub in $R = \frac{57Mg}{25}$, $T = \frac{6Mg}{5}$, $\sin\theta = \frac{4}{5}$

$$\mu = \frac{T\sin\theta}{R} = \frac{\frac{6}{5}Mg \times \frac{4}{5}}{\frac{57Mg}{25}} = \frac{8}{19} \quad (1)$$